THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR 3/15

THE MELBOURNE PROJECT

Students get creative and use mathematics skills to bring the sites of Melbourne into the classroom.

INSIDE

Teachers generally mention that they have very little time to deliver the entire curriculum. Whilst this is true, we need to consider the types of tasks we ask students to do. How can these encompass far more learning and teaching opportunities in comparison to a lesson where write on the white board and the children copy it down and then do some exercises.
The Melbourne Project enables the teacher to offer aspects of several subjects – mathematics, English, technology, and history all at once. This task can be as long or as short as you like.

WHAT IS IT?

Each student chooses a building in Melbourne (or relevant town) and researches its existence including its history, what it is made from, what currently happens there, how to get there, opening times, price etc. The student presents this in a tri fold brochure.
Pretty simple so far – unless you consider all the skills required to do this task.

Continued on page 6

FROM THE PRESIDENT

Jim Spithill - ACER

The 2015-2016 Council of the MAV met for the first time on June 4. The members represent a good cross section of the profession, from the classroom to academia. A report from CEO, Simon Pryor, listed more than thirty people who make MAV such an active organisation: volunteers including Council and committee members, the editors of our very informative journals, the Cliveden staff in administration, finance, community engagement and event management, and the very active education consultants who are the point of contact for planning and delivery of professional learning.

Council members act as convenors of the existing MAV committees: professional development, journals, resources, membership/marketing/communication, annual conference, student activities, life memberships and constitutional review. There is always scope for members to put a hand up to work in any of these areas, so feel free to contact Simon Pryor if you are interested. Committee members gain a good insight into the operation of the MAV.

Mathematics education seems to be in a constant state of flux and data from MAV member surveys in the last two years show that teachers are very concerned about the shortage of qualified mathematics teachers, posing a risk to the reputation and standing of the profession. Work by Paul Weldon at ACER quantifies the scale of the workforce challenges across education, with mathematics being one of the pressure points. Australian Bureau of Statistics population projections show a demographic change across Australia with a steep increase in primary school enrolments already happening and about to flow through to secondary schools from 2018. In Victoria alone the primary school student population is likely to increase by some 107 000 in the next ten years, or around 450 extra primary classes each year.

How to match supply and demand is a challenge. Where will the growth occur? Will it be sustained? There seems to be a sufficient supply of primary teachers coming through. However, at secondary level nearly half the males teaching mathematics are aged over 50 years. Already more than 20% of secondary mathematics classes are taken by out-of-field teachers. And 27% of primary and 20% of secondary teachers are part-time.

Aspects of a solution will include making it possible to upgrade the training of out-of-field teachers, increasing the retention rate of early career teachers, increasing the numbers and completion rates in pre-service mathematics courses, and taking in overseas-trained teachers. All of which requires a political commitment to making the teaching of mathematics an attractive and rewarding career.

Your MAV will continue to advocate for our profession to receive the resources it needs. Investment in teachers and teaching will underpin the response to these challenges.

- Jim Spithill

Reference

Weldon, Paul R. (March 2015). The Teacher workforce in Australia: Supply, demand and data issues. *Policy Insights*, Issue 2. Melbourne: ACER.

REVISION LECTURES

REGISTRATIONS FOR VCE STUDENT REVISION LECTURES NOW OPEN

Each year The Mathematical Association of Victoria presents VCE revision lectures for students in the three VCE Mathematics subjects: Mathematical Methods, Specialist Mathematics and Further Mathematics.

All lectures are delivered by highly qualified and experienced, practicing teachers.

Lectures will be held in the metropolitan and regional areas, in all regions.

To register go to http://registration.mav.vic.edu.au/mavrevlectures/.

For further information about the lectures contact Helen Haralambous hharalambous@mav.vic.edu.au

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MANAGING MATHS

Proudly supporting

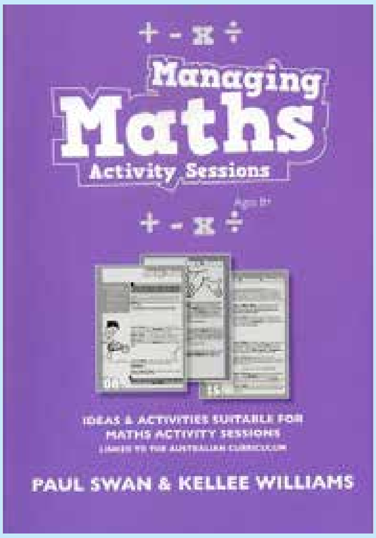
Breast Cancer Network Australia

The MAV are selling *Managing Maths Activity Sessions* by Paul Swan and Kellee Williams. The cost of this resource is $20 with $5 going directly to BCNA (Breast Cancer Network Australia).

Paul wrote this book with Kellee whose vocal cords were damaged as a consequence of radiotherapy. Kellee was unable to talk so writing this book was a terrifc way to keep her busy and motivated as part of her recovery. Sadly, Kellee passed away. In tribute to her memory, $5 from every sale of this book will be donated to BCNA.

This resource is ideal for National Literacy and Numeracy Week (31 August - 6 September). It's full of activities that teachers could run as a school maths day.

To order *Managing Maths Activity Sessions*, telephone 9380 2399 or purchase online at www.mav.vic.edu.au.



NATIONAL Literacy and Numeracy WEEK

THE MAGIC OF 1-9

Peter Maher - Mathematics Co-ordinator, Penleigh and Essendon Grammar School

The essence of a successful mathematics program lies in both its relevance and its ability to engage students. The activities found in this article are highly engaging and enable students to exercise their abilities to apply their acquired skills and concepts to unfamiliar situations, the true nature of problem solving.

The digits from 1 to 9 can be manipulated in a number of ways to produce solutions that appear to students to be almost 'magical'. Many of the activities found in this article have numerous possible answers ,that add to the mystique of the nature of our number system and, as a consequence, both challenge and delight. They encourage co-operative group work, (learning is essentially a social activity), and at the same time afford a 'low threshold and high ceiling' approach to differentiation.

ACTIVITY 1
THE 1- 9 MAGIC SQUARE

An excellent starting point is the well known magic square where students are asked to enter the digits from 1 to 9 into a 3 x 3 square in such a manner that the three rows, three columns and two diagonals all sum to the same total.

4	9	2
3	5	7
8	1	6

Why is five in the centre of the magic square? Are different solutions possible?

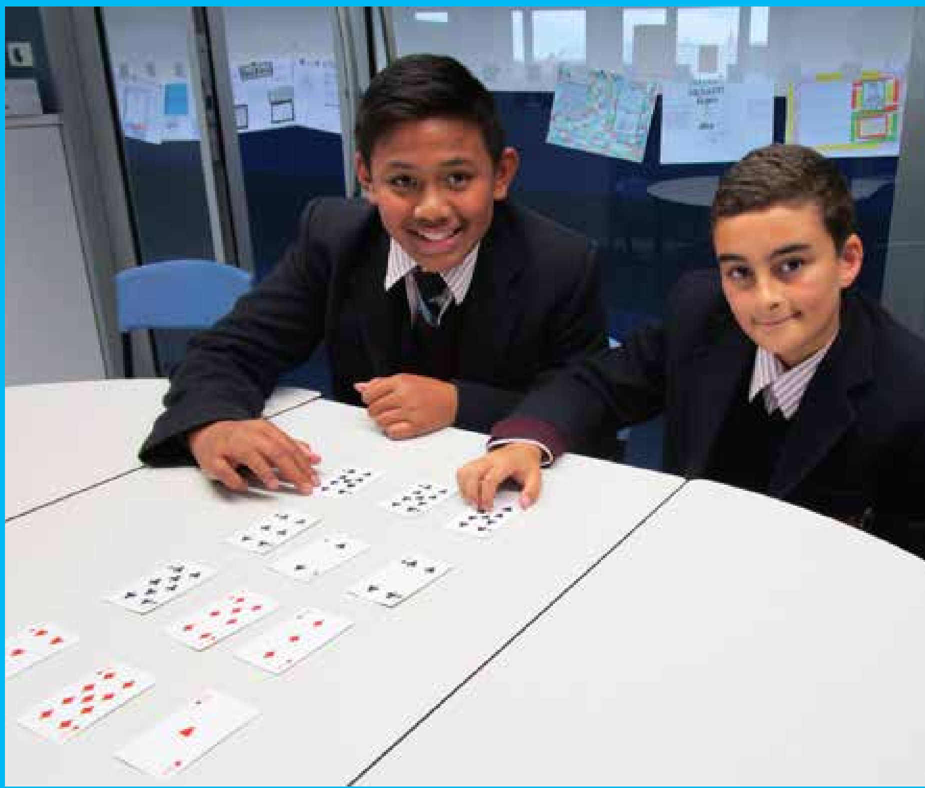
Can answers be found that do not have five in the centre?

ACTIVITY 2
MAKE 999

This exercise asks the students to use the digits from 1 to 9 to create three three-digit numbers that sum to 999. One possible solution is:

195
378
<u>426</u>
999

One of the most intriguing things about this task is the number of different solutions that work. Encourage students to check their answers with a calculator.



Make 999

Bear in mind, that in this instance, the calculator is not being used as the computational tool; its implementation is legitimised by being used to verify the students' work.

List the different solutions to enable the students to find patterns in the configurations that work, (where must the eight and the nine be found?). Remember that mathematics is basically the study of patterns and connections.

ACTIVITY 3
1-9 CARD CROSS

Ask the students to arrange number cards (or playing cards) into a cross in such a way that the two arms of the cross sum to the same value. One possible result is:

		9		
		7		
8	2	3	5	6
		1		
		4		

Are other results possible?

Must three be the pivotal point for the cross? Will the arms always sum to 24?

ACTIVITY 4
THE EINSTEIN PROBLEM

It has been suggested that at the age of six Albert Einstein was asked to arrange the digits from 1 to 9, in that order, to create an equation that equalled 100. Supposedly he wrote down
$1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100$.

Apocryphal or not, what has become known as the Einstein Problem is a wonderful way to exercise students' estimation skills and operational competence. What is rather amazing about this question is the fact that there are at least 10 other solutions that work. Perhaps the most elegant is $123 - 45 - 67 + 89$, requiring the use of only three operational signs. Once again, the students should be encouraged to check their estimates by using the calculator. However, a trap for young players is the use of the calculator to find a solution of 100 only to realise that the equation had not been written down first. Tantalisingly, there are numerous solutions equalling 99 and 101!

Due to the challenging nature of this task it is a worthwhile tip to give the less able mathematicians in the class a decent start, such as the first one or two steps contained within the equation.

Further extension and further very useful applications of the problem lie in the realm of decimals. Allow rational numbers to be incorporated within the equations and answers such as:

1.2 + 0.3 - 4 + 0.5 + 6 + 7 + 89

become possible. Again, as is the case with whole numbers, numerous solutions are possible when, if one pardons the dreadful pun, decimals are brought into the equation.

ACTIVITY 5
1 – 9 EQUALITY

This activity encourages students to consider fractions as ratios. Hopefully the days are now long gone when teachers advice students that to form an equal fraction 'you do to the top what you do to the bottom'. This classic rote oriented way of instruction may well result in the students getting a tick but will never result in a student developing a deep understanding of the nature of equal fractions. Yes, to form an equal fraction both the numerator and the denominator need to be multiplied by the same number, but these numbers must be seen as forming a fractional name for one whole number. Because the law of identity tells us that any number multiplied by one must in essence, remain the same, pairs of equal fractions fulfil the effects of this law.

Equal fractions can also be seen as demonstrating equal ratios.

$\frac{16}{32}$ and $\frac{4}{8}$ must be equal fractions

because the ratio of the numerators to the denominators is 1:2 or conversely 2:1 when the denominators are compared to the numerators.

This activity requires using the numbers from 1 to 9 to create a name for $\frac{1}{2}$.

Logic tells us that the fraction must contain four of the digits in the numerator and five in the denominator and that the denominator must start with one.

Card Cross

Common sense would seem to suggest that it would be remarkable to find an answer that works, like:

$$\frac{9273}{18546}$$

How truly amazing it is to be able to find multiple answers that have numerators starting with six, seven and nine. Pile on the wonder as your students come to the realisation that the structure of this question will lead to multiple fractional answers equalling

$\frac{1}{3}$ and $\frac{1}{5}$.

My students and I could only find one that equalled $\frac{1}{4}$ but there may be more.

CONCLUSION

Mathematics is a subject that contains a deal of inherent beauty. The activities outlined in this article will hopefully demonstrate this point to your students and at the same time inspire and challenge them to discover magical solutions.

REFERENCES

All You Need To Teach Calculators, Macmillan Education 2010

Macmillan Problem Solving Boxes 1 – 6, Macmillan Education, 2011

Maths Games On The Go, Macmillan Education, 2007

Teach Maths For Understanding – The Mathematical Association of Victoria

To share your innovative card games or for more ideas about how to use card and dice games to enhance your teaching and learning program contact MAV's mathematics education consultants on 9380 2399.

THE MELBOURNE PROJECT (CONT.)

Maria Harkins - St. Paul's Anglican Grammar School, Traralgon

WHERE IS THE MATHS?

The maths comes when we grid the floor with masking tape and create the map of Melbourne so that each student can create their own version of the building they have researched and then place it on the grid map in the appropriate reference point. Students assist in the creation of the grid after research and discussion as to how big the blocks in the CBD are and what size we should make ours. Other students then read the map and mark the names of the streets. We ensure that the compass points are also on the floor so that the children know which way we are standing when considering how to get to a certain venue. – This is only the first part that is related to maths.

After the brochure has been created and all research has been done, the student is then set to build their venue. They need to build it so that it can be placed on the map and be recognisable. A discussion is had regarding size. Scale is discussed but not strictly adhered to in regard to the height of those buildings with storeys. The buildings would be too small for the students to enjoy making, so we allow 3 cm per storey unless of course it is the Rialto or Eureka skydeck – these become 2 cm per storey for ease of making, and fitting it in the classroom. However, this is a shared understanding and the students know why the decision was made.

Next, the students set about making a design brief and then seeking materials if they need them. At this point, we accept and encourage their parents to get involved to assist them with whatever they need to create their building. Because we are assessing research skills and mathematical understanding, we are happy for the parents to assist as long as the student is able to explain exactly what they understand and how they achieved the created outcome. Students are aware that they will not be given a grade for their building so there is no issues with who had parental help and who didn't. Some students gain far more from their dad or mum helping them with ideas and technique, than years in the classroom ever could.

The variety of results is incredible. From shoe boxes and simple cardboard creations,

to electrical lighting, foam and weights, spinning wheels, wooden, clay, plaster, paper mache, printed facades, plastic, seeded grass, the options are endless. The students really enjoy this task.

When they bring in their creation, Melbourne comes together and it is magnificent. The most interesting part for me is that the students appreciate each others creation and want to show them off to their parents when they come in. Perhaps this is because there is no competition for grades.

Students need to deliver a spoken presentation to the class regarding their building.

They need to explain what it is and why a person would go there. They briefly speak about any difficulties they had in the build process and how they solved the problems.

Students and parents from the school community are encouraged to come and see Melbourne. The students are engaged to conduct tours where they explain what they know about their buildings.

Finally, the students are given the task of standing in the CBD at a spot on the map and creating instructions for someone else to follow to get to a certain destination. A person might begin at Federation Square, and give directions (head West along Flinders street etc) to get to Etihad stadium, but they do not say the destination. Then they swap directions and each has to identify the destination after they have read the map and followed the instructions.

The teacher then gives a reference point – corner of Spring Street and Bourke Street and the student notes what is at the point.

This part of the task can be as involved or as simple as you would like. My classroom is the old library so it is quite large and the carpet is old. If you don't have this as an option, perhaps the concrete outside could work for the grid. If it can't be permanent, then perhaps a projected picture onto the floor might work as well.

So, that is the Melbourne Project. I hope that you can see that it is the open ended process of this series of tasks which allows the greatest flexibility in the teaching, assistance and outcomes for the children.

TEDDY CARDS: COUNTING STRATEGIES FOR PREP

Rebecca Stokes - Deer Park North Primary School

I'm a second year graduate teacher and teach a class of sixteen Prep students. Fifty percent of the students are from EAL backgrounds, and just under a third of my class have high needs.

During the past year our school has been involved in coaching sessions with the MAV in collaboration with three other schools from the Western region. Within these sessions each school established their own goals, which became the focus of the coaching sessions.

Our goal at Deer Park North Primary School was to learn effective ways to enhance student engagement and create differentiated lessons to support all the students and their learning needs.

Based on the high level of EAL students in my class it is imperative to teach explicitly, whilst also structuring the lessons to cater for all the students and their needs. The coaching sessions with Ellen Corovic from MAV have been beneficial to me as a teacher to see how I can create lessons that are engaging, fun and inclusive for all the students, whilst also allowing students to work at their own level of learning.

Teddy Cards is a lesson I modeled to create engaging, differentiated lessons. This lesson was taught to Preps in Term 1. Prior to teaching this lesson we had explored counting forwards and backwards to ten, sequencing numbers in the correct order and creating quantities to ten.

During these lessons we had also explored what makes a good counter. This included adopting strategies such as pointing whilst counting, lining objects up, so that students could clearly see what needed to be counted and creating a starting and finishing point whilst counting.

At the beginning of this lesson we discussed what we had previously been learning in numeracy. Students came up with a variety of answers, such as sequencing numbers in order, matching numbers to the correct quantity and that we have been learning to be good counters. All of these answers made up the learning intention for this lesson.

LEARNING INTENTION

Today we are learning to match numbers to quantities and be good counters by using our counting strategies.

SUCCESS CRITERIA

We are successful when we can match numbers to the correct quantity.

LAUNCH

To launch this lesson we played musical numbers. Students were asked to dance around the classroom until the music stopped; when the music stopped I called out a number, which the students then had to create with the other students in the class.

During this activity students were asked to use their counting strategies to check they had the correct amount of people in their group.

We also discussed what they needed to do if they had too many or not enough people, such as, what do you think you need to do to create four? Do you need another person or do you need to politely ask someone to move to another group? I also questioned the students on how they knew they had the correct amount of people in their group? How do you know, how can you check? What do good counters do? What could you do to make it easier? Students used their counting strategies of pointing, lining students up and having a beginning and an end point whilst counting. After playing for a short period, students returned to the floor sitting in a circle.

Students were presented with the Teddy cards, which consisted of ten cards with the numbers from one to ten and ten teddy cards with subitised numbers from one to ten. (In the form of teddy bear pictures) Students were shown how some teddies had numbers and how some teddies had pictures. I explained that the aim of this game is to match the cards with numbers with the cards that have the same quantity of teddies. Such as, the teddy card with the number nine is a match with the teddy that has nine teddies.

Once we had looked at the teddy cards in detail I turned all the cards face down and asked a student to play with me. We took in turns of turning over two cards and asked different members of the class if the two cards presented were a match?

If the cards were a pair, the player who turned the matching cards got to keep them. The person who had the most cards at the end was the winner.

During this demonstration I displayed bad counting strategies, such as doubling up or matching the wrong cards. I did not tell the students I had done this on purpose. I waited for the students to recognise that I had counted the teddies incorrectly. When they identified my mistake, I asked them 'what should I do to count them correctly?' I then asked a student to model good counting strategies.

EXPLORE

Pairs of students worked together using a set of teddy number cards. Students being supported by their partner differentiated this part of the lesson; student's partners were established prior to the lesson. I also worked with a small group of students on the floor who needed extra support.

Students played this game a few times, before packing up students were asked to sequence all of their teddy number cards in the correct order. Before students could return to the floor I checked that each group had sequenced their cards in order. This enabled me to assess the student's knowledge of number sequencing and their ability to match the numbers with the correct quantity.

CONCLUSION

We discussed the lesson and some of the strategies the students had used used. As a reflection we discussed the importance of knowing all the aspects of a number, what a number looks like, what the numeral looks like and where it comes in sequence to the other numbers. To complete the lesson and again assess the student's knowledge of numbers and quantities we played another game of musical numbers. However, this time I did not call a number when the music stopped, I flashed a number card. Students had to read the number and sort themselves into groups based on their own perception of what the number was I had flashed.

To follow on from this lesson I continued looking at matching numbers to quantities and sequencing numbers in order. To differentiate the learning, I extended the students who were demonstrating good number knowledge to ten, whilst allowing other students to consolidate their number knowledge to ten. I established small groups, which enabled students to work at their own level, whilst still remaining connected to the main body of the lesson.

To learn about how a MAV Maths Education Consultant can create a specific coaching program for your school contact Jennifer Bowden, jbowden@mav.vic.edu.au.

SCHOOL OUTREACH PROGRAMS

FREE WORKSHOPS*

Years 9 to 10

MATHEMATICAL INVESTIGATION OF MUSICAL RHYTHMS

Find out the connection with musical rhythms and maths to solve problems. Look at maths differently by using combinations, geometry and number theory in an investigation of rhythm though classical music, rock, metal, alt, rap, Goth, world, folk as well as some forays into calendars, unending chess, neutron accelerators and origami.

GRAPH COLOURING

Mathematics is about solving problems. But you don't always have to do calculations, or use numbers. It is said that a picture is worth a thousand words. In this activity, you'll see pictures worth a thousand calculations – and perhaps create your own.

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DOES CHERYL DESERVE A BIRTHDAY PRESENT?

A tricky problem solving question went viral during April. This question was set as part of the Singapore and Asian Schools Maths Olympiads (SASMO). Give it a try!

Albert and Bernard just became friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

- *May 15*
- *May 16*
- *May 19*
- *June 17*
- *June 18*
- *July 14*
- *July 16*
- *August 14*
- *August 15*
- *August 17*

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard does not know too.

Bernard: At first I don't know when Cheryl's birthday is, but I know now.

Albert: Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

To find the methodology and the answer, visit, www.dailymail.co.uk/sciencetech/article-3037266/The-maths-problem-set-Singapore-teenagers-left-people-world-stumped.html#ixzz3aGKPVIOS.

LEAVE SCHOOL

We all know how hard it is to have a productive meeting at school. Interruptions, emails and other distractions mean that meetings aren't as effective as they could be. Do you want an alternative venue - away from school - for your maths faculty staff meeting or for the mathematics component of a school Professional Learning Day?

Do you want to incorporate reviewing of resources without having to request inspection copies of a number of resources to be delivered to your school?
The MAV can help!

MAV CAN HOST YOUR NEXT MATHS STAFF MEETING OR PROFESSIONAL LEARNING DAY

The MAV can host your staff meeting, netwok or cluster meeting. MAV member schools can book the MAV's library or education centre to run after school maths staff meetings or PLT sessions.

Full access to the MAV library and bookshop is available on the day. One of our mathematics consultants can show you through maths resources or have a chat about what is happening at your school.

WiFi, tea and coffee are provided. Catering can be arranged if required.

For more information contact Darinka Rob, drob@mav.vic.edu.au or telephone 9380 2399.

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- Malvern Central School
- Milgate Primary School
- Mount View Primary School
- Penleigh and Essendon Grammar School (Primary)
- Richmond West Primary School
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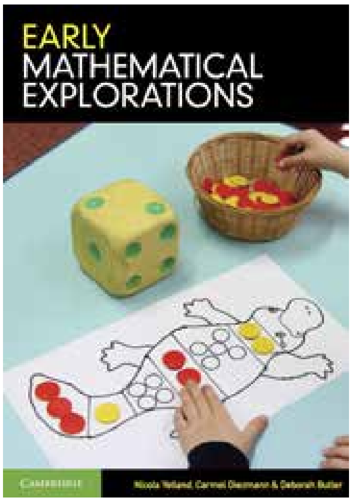
- Serpell Primary School
- Springside College
- Stonnington Primary School
- St Thomas Moore Primary School
- Sunshine College

To find out how your school can gain this accreditation, visit www.mav.vic.edu.au.

BOOK REVIEW
EARLY MATHEMATICAL EXPLORATIONS

Ivanka Vinski - Kinder to Prep Transition Coordinator, Deer Park North Primary School



As a Prep teacher, I am always looking for ideas for great numeracy lessons that are engaging, rich and able to be easily applied in my classroom for a wide range of learning abilities. I have recently found a book that is very useful and full of great lesson ideas. The book, *Early Mathematical Explorations* by Nicola Yelland, Carmel Diezmann and Deborah Butler, gives teachers rich and engaging activities so students can explore concepts and build on existing knowledge.

NUMBER MONSTER

I used a lesson idea from the book, *Number Monster* and implemented it in my classroom. I wrote a number on the back of a paper plate and asked the students to find out what their number was. That was their first challenge. I gave the students individual numbers that I knew would be slightly challenging for them. The students needed to give their monster features using their number. So if their number was five, they needed to give their monster five of everything! The students loved the lesson! It was a great hands-on activity that gave multiple opportunities for the students to apply their number knowledge to create their number monster. The lesson allowed for student differentiation and was easy to modify when it became too easy or difficult for the students.

By changing the number for each student, there was a sense of excitement that they are working on a special number that no one else has. They cannot then 'copy' the work of another student, they are required to apply their knowledge and thinking to complete the task.

BENEFITS OF THE LESSON

- There was scope for explicit teaching; counting with 1:1 correspondence to match their given number
- Students were challenged by providing a number appropriate to their mathematical development;
- It provided a base for good questions. eg., asking students to check their number and if they had enough, too much or too little and what they would do to make their monster's features match their number
- Extend students by asking them to add more or take away when they had too many or too little; and
- Extending students by giving a second larger number that they needed to amend their monster to suit.

The students were highly engaged throughout the lesson, and although messy at times (like many Prep classrooms) the students thoroughly enjoyed using crafty materials to create their monsters. They are proudly displayed in the classroom and the students were able to show how they represented their number by their monster's features and explain how they made their unique number, by counting and giving the number of the groups, or by telling a story of how they had too many or too little and had to add or take away.

The activity also provided avenues to take the lesson idea into future mathematics lessons to continue exploring numbers and making groups to match. Further follow up lessons incorporating this number making concept included dividing a page into four boxes, writing a number in the middle and asking the students to find out their number and show that number four different ways by using concrete materials. The students were again very engaged during this lesson and were able to apply their knowledge to demonstrate their number knowledge. By asking students to create the same number more than once, it provides for lots of counting practice and using concrete materials helps to deepen understanding of particular numbers.

Some students were having difficulty counting beyond a particular number and this lesson provided a chance to scaffold learning by helping to bridge the gap between the number students could count to and the number that came next.

It is not as difficult as it may seem to plan fun, hands on and creative lessons that allow for student differentiation and targeted learning. By using lessons ideas such as the ones mentioned, it can be easy for the teacher to differentiate the learning for the students, and provide individual learning goals and challenges in a fun and creative way.

Early Mathematical Explorations is available in the MAV's online shop.
Member price $65.32.
visit www.mav.vic.edu.au/shop or telephone 9380 2399.

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MATHEMATICA AND THE AUSTRALIAN CURRICULUM

Carmen Popescu-Rose, Director, Mathematics for Excellence. www.wolfram.com/training/instructors/popescu-rose.html.

The purpose of this article is to provide a list of Mathematica functions that could be used in teaching and exploring Graphs, Networks and Decision Mathematics as outlined in the Australian Curriculum at Year 12 level for General Mathematics [1]. While Mathematica is a powerful tool in analysing, modelling and structuring graphs and networks, this article should be seen as a basic introduction into this functionality for the novice user.

CONSTRUCTING SIMPLE GRAPHS AND NETWORKS

The two main elements required to construct a graph are *vertices* or *nodes* and *edges* connecting these vertices.

A graph, in Mathematica, can be generated by using the function Graph[ ]. Other functions that can be used to generate graphs and networks are GraphPlot[ ], GraphPlot3D[ ], LayeredGraphPlot[ ]and TreePlot[ ].

SIMPLE GRAPHS

The operator used to represent an undirected edge of a graph is •—• and can be typed in using esc ue esc. A simple undirected graph connecting three nodes, A, B and C, Figure 1(a), and the corresponding command, Figure 1(b) are shown. VertexLabels is an option that specifies the labels to be used for vertices.

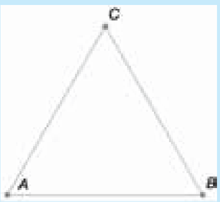


Figure 1(a). A simple undirected graph.

```
Graph[{
  A ⟷ B,
  A ⟷ C,
  B ⟷ C},
 VertexLabels → "Name"]
```

Figure 1(b). Mathematica command for the simple undirected graph.

The default operator used for a directed edge is → and can be typed in using esc -> esc. The operator •→ can be also used for this purpose and can be typed in using esc de esc.

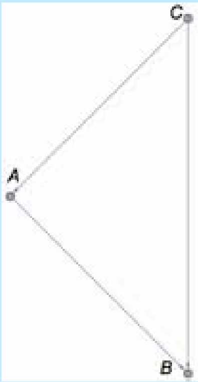


Figure 2(a). A directed graph.

```
Graph[{
  A → B,
  C → A,
  C → B},
 VertexLabels → "Name"]
```

Figure 2(b). Mathematica command for the directed graph.

The GraphPlot[ ] function is very similar to the Graph[ ] function; however, directed graphs need the option DirectedEdges->True in order to have the edges' directions displayed and VertexLabeling->True for vertices to be labelled (Figure 3).

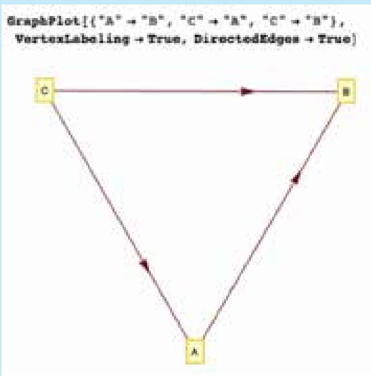


Figure 3(a). Directed graph generated with the GraphPlot[ ] function with labels for vertices and directions on edges.

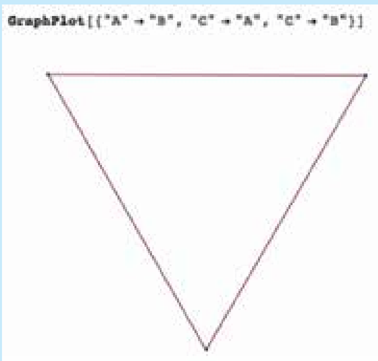
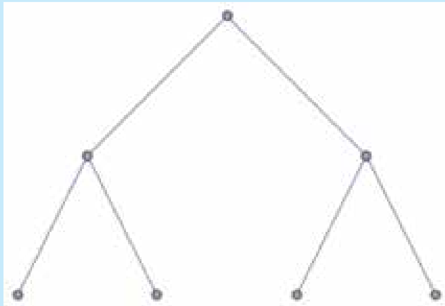


Figure 3(b). Directed graph generated with the GraphPlot[ ] function without labels for vertices and no directions on edges.

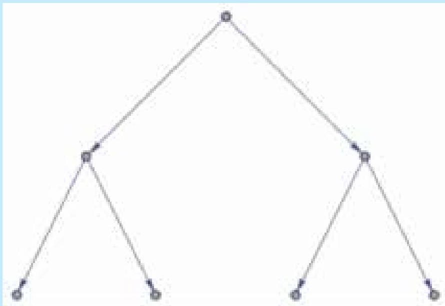
TREE GRAPHS

Whether a tree graph is directed or undirected, the command used to produce it is the same. The only difference is in the operators used: •—• for undirected edges and → or •→ for directed edges.



```
TreeGraph[{
  1 ⟷ 2, 1 ⟷ 3,
  2 ⟷ 4, 2 ⟷ 5,
  3 ⟷ 6, 3 ⟷ 7}]
```

Figure 4(a). Undirected tree graph with the corresponding Mathematica command.



```
TreeGraph[{
  1 → 2, 1 → 3,
  2 → 4, 2 → 5,
  3 → 6, 3 → 7}]
```

Figure 4(b). Directed tree graph with the corresponding Mathematica command.

A second function that could be used to generate tree graphs is the TreePlot[ ] function with (Figure 5 (a)) and without (Figure 5 (b)) labels for vertices and no directions on edges.

```
TreePlot[{1→2, 1→3, 2→4, 2→5, 3→6, 3→7},
  VertexLabeling→True,
  DirectedEdges→True]
```

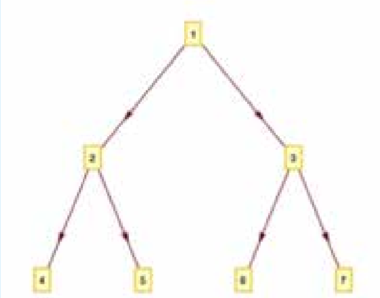


Figure 5(a). Directed graph generated with the TreePlot[ ] function with labels for vertices and directions on edges.

```
TreePlot[{1→2, 1→3, 2→4, 2→5, 3→6, 3→7}]
```

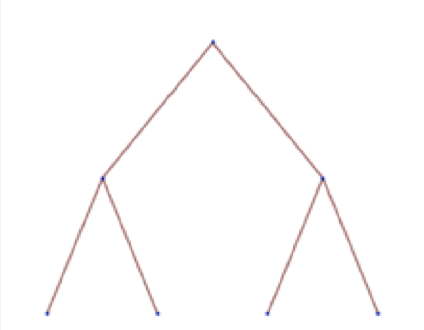


Figure 5(b). Directed graph generated with the TreePlot[ ] function without labels for vertices and no directions on edges.

BIPARTITE GRAPHS

A bipartite graph is automatically drawn in two rows using the same command as for the simple graphs. The bipartite graph in Figure 6 displays games played between six AFL teams. This diagram shows three away teams (Melbourne, Carlton and the Swans) who played against three home teams (Crows, Tigers and Dockers).

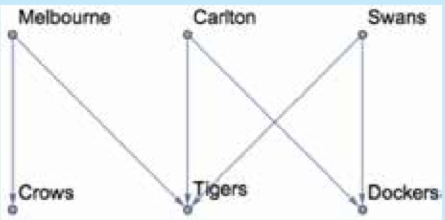


Figure 6 (a). Bipartite graph with directed edges d.

```
Graph[{
   "Carlton" → "Tigers",
   "Melbourne" → "Crows",
   "Swans" → "Dockers",
   "Carlton" → "Dockers",
   "Swans" → "Tigers",
   "Melbourne" → "Tigers"},
  VertexLabels → "Name"]
```

Figure 6 (b). The Mathematica command.

ADJACENCY MATRIX AND GRAPHS

The graph in Figure 7 displays a network of six people sending emails to each other. The loop represents the fact that Carol is in the habit of forwarding important emails to herself as a reminder.

```
g = Graph[{"Alana" ↔ "Carmen",
   "Carmen" ↔ "Carol",
   "Carol" ↔ "Vanessa",
   "Alana" ↔ "Vanessa",
   "Vanessa" ↔ "Carol",
   "Carol" ↔ "Carol",
   "Carmen" ↔ "Ying",
   "Ying" ↔ "Alana",
   "Ying" ↔ "Vlad",
   "Carmen" ↔ "Vlad"},
  VertexLabels → "Name"]
```

Figure 7(a). Input for displaying the exchange of emails between six people.

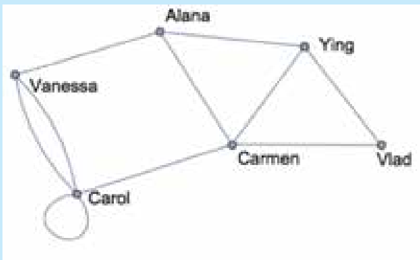


Figure 7(b). Output for displaying the exchange of emails between six people.

The adjacency matrix for this graph can be displayed by using the command AdjacencyMatrix[%] // MatrixForm, where [%] is the last result generated (the graph). Vertices are displayed in the same order as they appear in the input. The first row and column display the number of emails exchanged between the first person (Alana) and the others. The second row and column display the emails exchanged between the second person (Carmen) and the others. Moving along the input of the graph, the next people, in order of appearance, are Carol, Vanessa, Ying and Vlad.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Figure 8. The corresponding adjacency matrix of the graph from Figure 7.

The same Matrix can be displayed in table form with headings for easier reading and interpretation. The graph has already been defined with the command g=Graph[ ]. The list of names to be used as headings has to be defined also by using the command name={Alana,Carmen,Carol,Vanessa,Ying, Vlad}.

```
TableForm[AdjacencyMatrix[g],
 TableHeadings → {name = Style[#, {Bold, Magenta}] & /@ %, name}]
```

	Alana	Carmen	Carol	Vanessa	Ying	Vlad
Alana	0	1	0	1	1	0
Carmen	1	0	1	0	1	1
Carol	0	1	1	2	0	0
Vanessa	1	0	2	0	0	0
Ying	1	1	0	0	0	1
Vlad	0	1	0	0	1	0

Figure 9. Input and output for the table form of the matrix with headings.

The new symbols used in Figure 9 (b) are:

- # (slot) is short for #1. In this example, # represents the headings of each row/ column.
- & (logical AND). When applied as one symbol (&) it takes into account both expressions on either side.
- /@ (map) applies the font style to each element in the list.
- % last output.

An adjacency matrix can be also displayed for a directed graph. If in the previous example the paths of the emails are shown as directed edges (from the sender to the receiver), the graph is a directed graph as shown in Figure 10.

```
g = Graph[{"Alana" → "Carmen",
   "Carmen" → "Carol",
   "Carol" → "Vanessa",
   "Alana" → "Vanessa",
   "Vanessa" → "Carol",
   "Carol" → "Carol",
   "Carmen" → "Ying",
   "Ying" → "Alana",
   "Ying" → "Vlad",
   "Carmen" → "Vlad"},
  VertexLabels → "Name",
  VertexLabelStyle → Directive[Black, 15]]
```

Figure 10(a). Input for the emails example as a directed graph.

MATHEMATICA AND THE AUSTRALIAN CURRICULUM (CONT.)

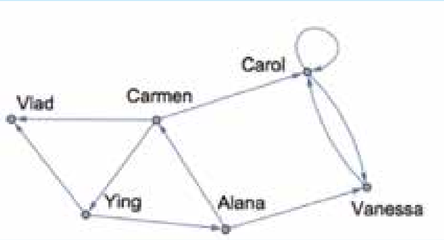


Figure 10(b). Output for the emails example as a directed graph.

The adjacency matrix for the directed graph is shown in Figure 11. The order of the vertices is the same as in the undirected graph: Alana, Carmen, Carol, Vanessa, Ying and Vlad.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 11 The corresponding adjacency matrix of the directed graph from Figure 10.

If the adjacency matrix is known, Mathematica has the functionality of converting the matrix into a graph. A symmetrical matrix generates an undirected adjacency graph (Figure 12) while an unsymmetrical matrix generates a directed adjacency graph (Figure 13).

$$\texttt{AdjacencyGraph}\left[\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}\right]$$

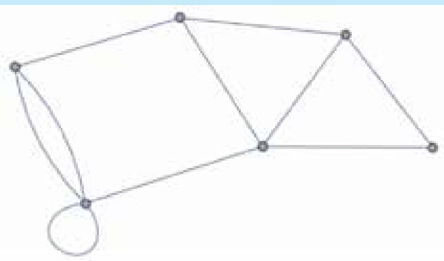


Figure 12. The corresponding adjacency graph (bottom) from a symmetrical matrix (top).

```
AdjacencyGraph[{
  {0, 1, 1, 0},
  {1, 1, 0, 1},
  {1, 0, 1, 0},
  {0, 1, 0, 1}}]
```

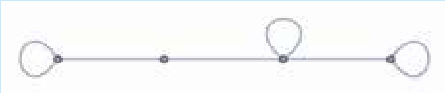
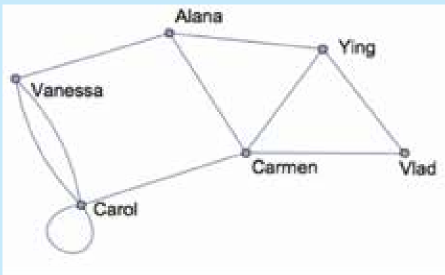


Figure 13. The corresponding adjacency graph (bottom) from an unsymmetrical matrix (top).

3 PATHS AND CYCLES

Any graph can be tested to check whether it is an Eulerian graph, a Hamiltonian graph or neither. The undirected graph for the example with the exchange of emails between the six people is Eulerian but not Hamiltonian. The commands used to check the graphs for these properties are EulerianGraphQ[graph] and HamiltonianGraphQ[graph]. If the graph is either Eulerian or Hamiltonian, the output will be True and if it isn't, the output will be False.

Let us consider the exchange of emails example. The graph is not an Eulerian graph; however, it is a Hamiltonian graph (Figure 14).




```
HamiltonianGraphQ[g]
True
```

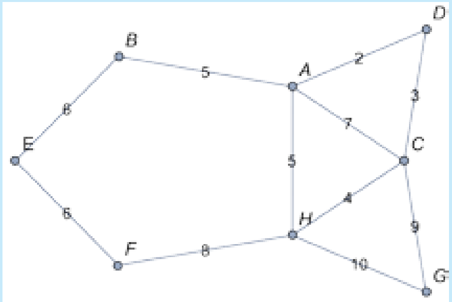
```
EulerianGraphQ[g]
False
```

Figure 14. (top) Hamiltonian graph (middle) Test for Hamiltonian graph (bottom) Test for Eulerian graph.

This graph can be further analysed for Hamiltonian cycles. To find a Hamiltonian cycle the function is FindHamiltonianCycle[g] with the output

```
{{"Alana" ↔ "Vanessa",
  "Vanessa" ↔ "Carol",
  "Carol" ↔ "Carmen",
  "Carmen" ↔ "Vlad",
  "Vlad" ↔ "Ying",
  "Ying" ↔ "Alana"}}
```

The following example is an Eulerian graph with weighted edges. The command for generating this graph includes the weights of all edges. Edges and vertices don't have to be labelled; however, it is more clear when labels are displayed on the graph.




```
g = Graph[{
    A ↔ B, A ↔ C, C ↔ D, A ↔ D,
    B ↔ "E", C ↔ G, C ↔ H, G ↔ H,
    H ↔ F, F ↔ "E", A ↔ H},
   EdgeWeight → {5, 7, 3, 2, 6, 9, 4, 10, 8, 6, 5},
   EdgeLabels → {
     A ↔ B → 5, A ↔ C → 7, C ↔ D → 3, A ↔ D → 2,
     B ↔ "E" → 6, C ↔ G → 9, C ↔ H → 4, G ↔ H → 10,
     H ↔ F → 8, F ↔ "E" → 6, A ↔ H → 5},
   VertexLabels → "Name"]
```

Figure 15. (top) Weighted Eulerian graph, (bottom) Corresponding input.

A similar command that was used to generate a Hamiltonian cycle in the previous example can be used to generate an Eulerian cycle: FindEulerianCycle[g]. This command generates the Eulerian cycle

```
{{A ↔ D, D ↔ C, C ↔ H, H ↔ G, G ↔ C,
C ↔ A, A ↔ H, H ↔ F, F ↔ E, E ↔ B, B ↔ A}}
```

To generate a given number n of Eulerian cycles, the command changes to FindEulerianCycle[g,n] or FindEulerianCycle[g,All] for all Eulerian cycles of a graph.

The Eulerian cycle can also be highlighted edge-by-edge displaying each step in a table as shown in Figure 16 using the command

Table[HighlightGraph[g,Part[First[%],1;;i]],{i,Length[First[%]]}].

Note: ;; represents a span from 1 to i.

```
Table[HighlightGraph[g, Part[First[%], 1 ;; i]], {i, Length[First[%]]}]
```

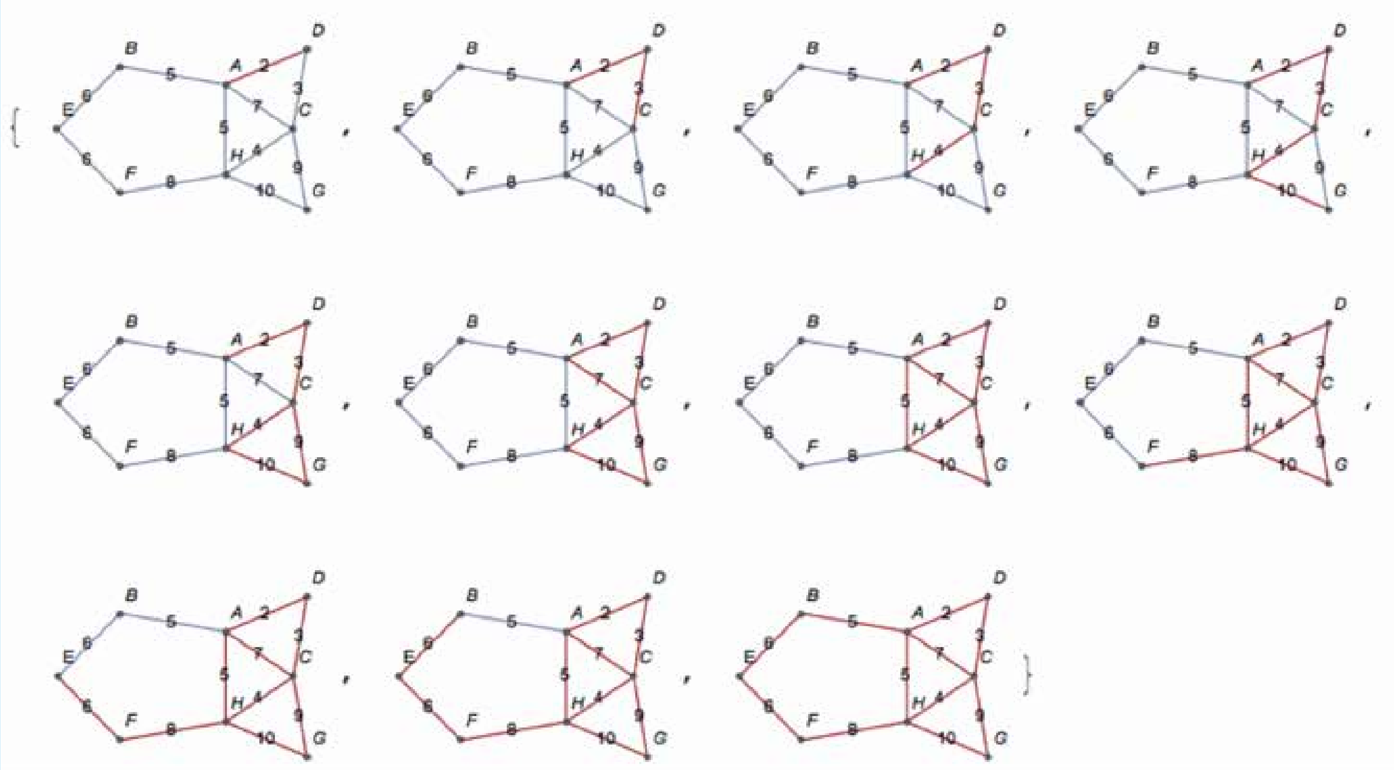


Figure 16. Highlighted Eulerian cycle displayed in a table.

SHORTEST PATH

The shortest path in a graph is generated by the command FindShortestTour[g]. For the graph discussed so far, the output generated for the shortest path is {49,{A,D,C,G,H,I,E,B,A} and it is highlighted as shown in Figure 17.

```
HighlightGraph[g, PathGraph[%[[2]]]]
```

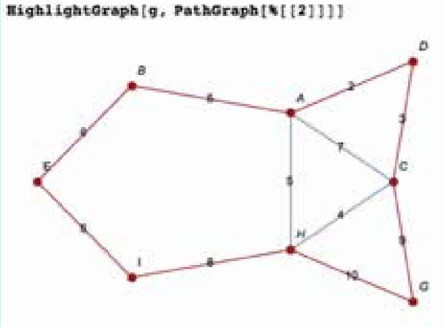


Figure 17. Highlighted shortest path of the graph from Figure 15 (top) with the corresponding input.

SPANNING TREES

The spanning tree for the graph g can be defined using st=FindSpanningTree[g]. This command will generate the spanning tree for the graph g as shown in Figure 18.

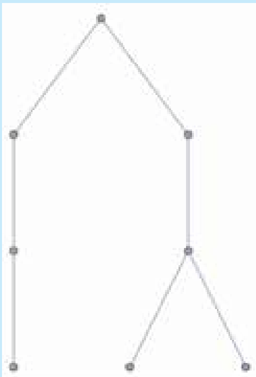


Figure 18. The spanning tree for the graph from Figure 15.

```
HighlightGraph[g, st, GraphHighlightStyle → "Thick"]
```

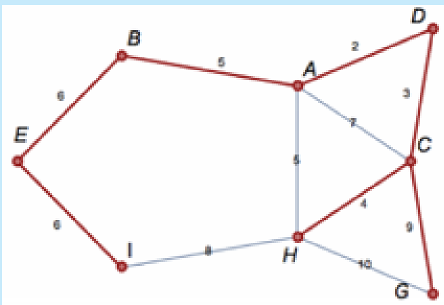


Figure 19. (top) Input to highlight the spanning tree on the graph. (bottom) Highlighted spanning tree.

In a similar way, the shortest path between two nodes, for the first example, can be generated using FindShortestPath[g,Vanessa,Vlad] with the output {Vanessa,Alana,Carmen,Vlad} and the path highlighted in Figure 20 (bottom).

```
HighlightGraph[g, PathGraph[%]]
```

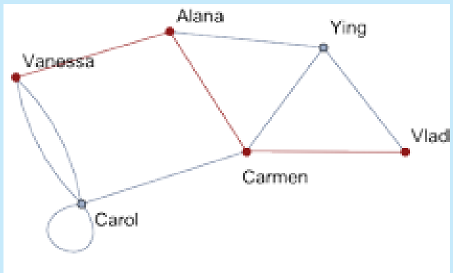


Figure 20 (top) Input to highlight the shortest path on the graph. (bottom) Highlighted shortest path

MATHEMATICA AND THE AUSTRALIAN CURRICULUM (CONT.)

FIND MINIMUM CUT

Another Mathematica functionality is the ability to determine the minimum cut of a graph.

```
FindMinimumCut[g]
{5, {{A, B, C, E, G, H, I}, {D}}}
```

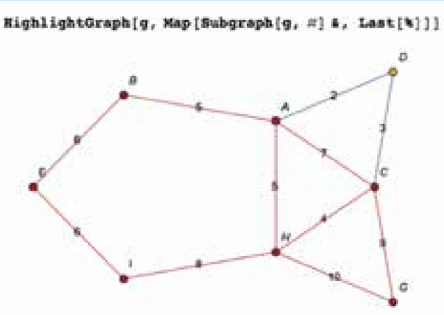


Figure 21 (top) Input to highlight the shortest path on the graph. (bottom) Highlighted shortest path.

The Wolfram Demonstrations Project is a Wolfram Mathematica powered site with over 9800 interactive projects contributed by Mathematica users from all over the world.

The following interactive demonstration is a very well known problem, *The Seven Bridges of Königsberg* solved by Leonhard Euler. Students could use this activity to further their understanding of graph theory.

All the functions and operators presented in this article are just a small part of the Mathematica functionality for graphs and networks. There is a countless number of resources at the Wolfram website that could be further explored.

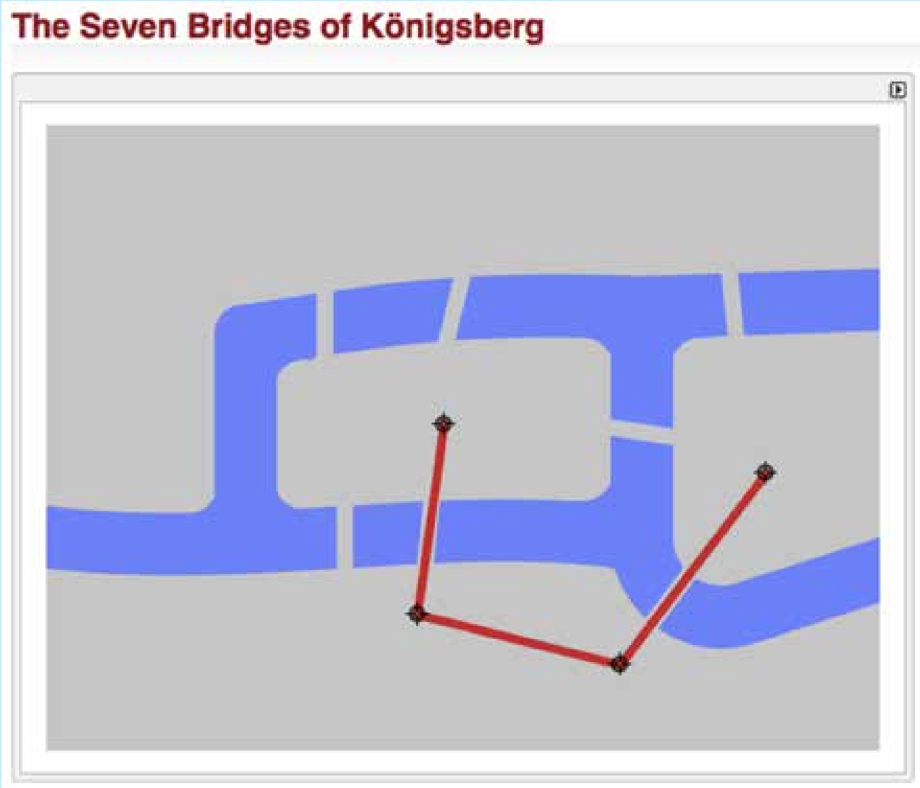


Figure 22. Interactive demonstration "The Seven Bridges of Königsberg", http://demonstrations.wolfram.com/TheSevenBridgesOfKoenigsberg/. Wolfram Demonstrations Project . Contributed by S. M. Blinder.

REFERENCES

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Wolfram Mathematica Tutorial collection, Graph Drawing

www.wolfram.com/learningcenter/tutorialcollection/GraphDrawing/GraphDrawing.pdf

S. M. Blinder "The Seven Bridges of Königsberg"

http://demonstrations.wolfram.com/TheSevenBridgesOfKoenigsberg/

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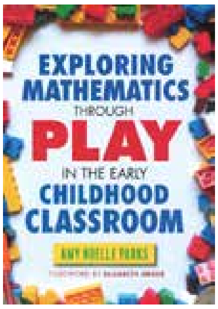


EXPLORING MATHEMATICS THROUGH PLAY

EC

This practical book provides teachers with an understanding of how maths can be learned through play. The author helps teachers to recognise the mathematical learning that occurs during play, to develop strategies for mathematising that play, and to design formal lessons that make connections between mathematics and play. Classroom examples illustrate that, unlike most formal tasks, play offers children opportunities to solve non-routine problems and to demonstrate a variety of mathematical ways of thinking - such as perseverance and attention to precision.

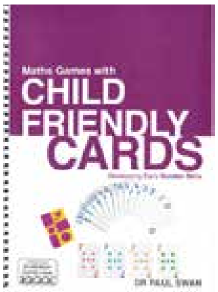
$62.40 (MEMBER)
$78 (NON MEMBER)

COMBO CARD GAME

5-8

COMBO is featured in Dr Paul Swan's book *Cards on the Table*. The game is designed to help students practice their basic number facts. An ideal game for children in upper primary and lower secondary school. Each set contains: 1 set of yellow Combo cards and 1 set of green Combo cards.

$5.72 (MEMBER)
$7.15 (NON MEMBER)



MATHS GAMES WITH CHILD FRIENDLY CARDS BY DR PAUL SWAN

F-4

This book will give you plenty of ideas to use Child Friendly Playing Cards for a wide range of mathematical activities. Playing cards are an extremely versatile resource to re-inform many key maths concepts and this book will help teachers to assist young children in learning these key early number concepts while playing simple card games. The child friendly playing cards display numbers 0-13 instead of the Jack, Queen, King and Ace, and comes in 4 colours and shapes instead of the suits. This completely removes the gambling association that often comes with playing cards.

$21.45 (MEMBER)
$26.81 (NON MEMBER)

CHILD FRIENDLY NUMBER PLAYING CARDS

F-6

These playing cards have been designed for younger children. The face cards are replaced with 0, 11, 12 and 13 and the suits replaced with triangles, rectangles, squares and circles. These shapes are arranged in standard subitising patterns. Each set contains: 1 set of blue Child Friendly cards and 1 set of purple Child Friendly cards.

$5.72(MEMBER)
$7.15 (NON MEMBER)

ROWCO CARD GAME

4+

ROWCO is also featured in *Cards on the Table*. This game links basic additions and subtraction facts with problem solving and reasoning. This game is suitable for children from Year 4 onwards. Each set contains 1 set of blue ROWCO cards and 1 set of red ROWCO cards.

$5.72(MEMBER)
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